

# Estimation of the Dispersion Coefficient with Acoustic Doppler Current Profiler (ADCP) at Barak River

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**Abstract**—Estimation of longitudinal dispersion coefficient ( $K_x$ ) is made from the flow velocity and river cross sectional geometry measured with Acoustic Doppler Current Profiler (ADCP). Relative influence of different parameters on dispersion coefficient ( $K_x$ ) is also evaluated through sensitivity analysis. Normally the dispersion coefficient is measured by costly and time consuming tracer studies because the velocity field cannot be resolved sufficiently before the flow changes. However, ADCP transects, which are used to measure discharge, provide detailed velocity and bathymetry data quickly. In the present study an empirical equation derived from Fischer's (1967) triple integral expression for natural rivers is used for determining the longitudinal dispersion coefficient from measured field data of river Barak. The method is based on the hydraulic geometry relationship for stable rivers and on the assumption that the uniform-flow formula is valid for local depth-averaged variables. From the sensitivity analysis, it is found that the average velocity is the most important parameter controlling the dispersion coefficient. The result of the present study will be useful in identifying water intake and pollutant disposal point.

**Keywords:** Acoustic techniques, Bed shear velocity, Dispersion coefficients.

## 1. INTRODUCTION

Predicting the spread of pollutants is important for managing and protecting rivers and streams. To simulate contaminant dispersion, most mixing models require a longitudinal dispersion coefficient, which depends on the river geometry and flow. The dispersion coefficient has generally been estimated with empirical formulas or costly field tracer experiments. Tracer studies also require a large investment in planning, staff, and analysis. Another approach to determining the longitudinal dispersion coefficient ( $K_x$ ) is to estimate it directly from the theory of shear dispersion with (Fischer 1967)

$$K_x = -\frac{1}{A} \int_0^B hu'dy \int_0^y \frac{1}{Dh} dy \int_0^y hu'dy \quad \text{-- (1)}$$

where  $y$  = transverse coordinate that runs from 0 at one bank to the width  $B$  at the other;  $A$  = cross-sectional area;  $h$  =

depth;  $D$  = transverse mixing coefficient;  $u' = u - U$  = velocity deviation;  $u$  = depth-averaged stream-wise velocity; and  $U$  = velocity averaged over the cross section. Eq. (1) has been employed as the basis of various empirical methods determining the longitudinal dispersion coefficient. However, there is a misconception concerning parameter  $D$ .

Use of Eq. (1) is limited by the assumptions in the theory. In particular, the flow is assumed to be one-dimensional; that is, the contaminant must be well mixed in the transverse and vertical directions. This assumption may limit the validity of Eq. (1) in regions where the contaminant is not well mixed; in recirculation zones, which prevent the decay of the concentration profile to a Gaussian profile; and at bends, where strong secondary currents are present (Fischer 1969). Eq. (1) also requires the width of the river to be much larger than the depth, so that the transverse shear, and not the vertical shear, controls the dispersion. Further, the transverse velocity gradient must be large enough for shear dispersion to dominate over other spreading mechanisms. This assumption may fail in slowly moving reaches of a river, such as those with recirculation zones, or in rivers where other mechanisms may be important. Here, both the local flow depth  $h(y)$  and the deviation  $u'(y)$  of the local velocity from the cross-sectional mean velocity are defined based on the straight symmetrical channel and the uniform flow. However,



**Fig. 1: The nine measured transects shown from Google map. The corresponding numerical digits indicate transect number.**

natural rivers involve many kinds of non-uniformities, such as dead zones, bends, and islands (Sooky 1969; Rutherford 1994). There even exists secondary flow in straight natural rivers (Nezu et al. 1993). These non-uniformities of channel geometry and flow affect the theoretical definitions of  $h(y)$  and  $u'(y)$  and thus the dispersion coefficient  $K_x$ . It is therefore newly introduced to account for the various non-uniformities involved in both flow and geometrical characteristics of natural rivers. Some experimental results provide quantitative information about the influence of non-uniformities.

Fischer (1967) conducted a series of dispersion experiments in laboratory channels with smooth and rough banks. The two sets of experiments were purposefully made under conditions as nearly identical as possible, except for the bank roughness. The experimental results show that the longitudinal dispersion coefficient in the channel with rock sides was about 15 (14.7) times that in the smooth channel. Most of the real channels possess rough sides due to the bank vegetation, groins, irregular bank alignment formed by natural erosion of flow, and other non-uniformities mentioned earlier. Considering all the factors to make applicable to natural rivers and streams, he developed the following equation:

$$\frac{K_x}{Hu^*} = \frac{0.15}{8D} \left(\frac{B}{H}\right)^{\frac{5}{3}} \left(\frac{U}{u^*}\right)^2 \text{ ----- (2a)}$$

Or,

$$K_x = \frac{0.15}{8D} \left(\frac{B}{H}\right)^{\frac{5}{3}} \left(\frac{U}{u^*}\right)^2 Hu^* \text{ ----- (2b)}$$

Eq. (2b) stems from the direct integration of (1) and is thus theoretically based. Moreover, Eq. (1) not only includes the conventional parameters, channel width-to-depth ratio  $B/H$ , and friction term  $U/u^*$  but also involves the effect of transverse mixing  $D$ . This distinguishing feature of (2b) is that it clarifies its dispersion mechanism. In addition, (2b) is conducive to further improvement if a more accurate transverse dispersion equation is found.

**2. LOCALITY AND SITE DESCRIPTION**

The Barak River is one of the major rivers of South Assam (India). It originates from Manipur hills at an altitude of 3015m, after Manipur it flows through Mizoram and then into Assam. It flows west past the town of Silchar where it is joined by the Madhura River. After Silchar, it flows for about 30 kms and it enters Bangladesh.

The drainage area of the sub-basin lying in India is 41,157 sq. km. In the Barak valley the width of the river varies from 120 to 250m and the bed gradient is very flat varies from 1:10,000 in the upper reach to 1:20,000 in the lower reach. The culturable area in the sub-basin 0.893 M-ha which is about 0.5% of the culturable area of the country. The river is the main source of water for irrigation, public water supply and

hydropower generation. The average annual surface water potential of the valley is 48.4 km<sup>3</sup>. So,

**Table 1: River properties and Dispersion Coefficients calculated from the Acoustic Doppler Current Profiler (ADCP) measurement**

Transect No	Width B (m)	Average depth H (m)	Average Velocity U (m/s)	Shear velocity $u^*$ (m/s)	Transverse mixing coefficient D (m <sup>2</sup> /s)	Longitudinal Dispersion coefficient $K_x$ (m <sup>2</sup> /s)
1	150.51	1.927	0.510	0.108	0.138	187.943
2	184.624	5.861	0.127	0.099	0.109	29.985
3	150.708	7.119	0.139	0.098	0.120	24.837
4	155.73	5.686	0.142	0.088	0.095	32.125
5	110.18	2.553	0.472	0.113	0.104	139.528
6	124.07	2.346	0.474	0.101	0.110	157.304
7	99.31	6.620	0.191	0.089	0.101	27.153
8	134.93	3.111	0.302	0.098	0.093	95.541
9	102.35	5.787	0.204	0.088	0.091	34.404

knowledge of pollutant content in this river water is very important feature.

Our area of measurement of discharge is at near Silchar town (Fig. 1).The study area cover almost a half wave-length the meandering channel. We have collected data from nine transects at different bend angles with ADCP mounted from moving boat.

**3. METHODOLOGY**

The ADCP used in the measurements reviewed here were downward looking; SonTek ADCP M9 with frequencies of 3 MHz. The ADCP measures the velocity of the water at several points along the all three coordinate directions. The ADCPs were the water at several points along the all three coordinate directions. The ADCPs were mounted to boats or towed on lines across the rivers to provide detailed measurements of vertical profiles of velocity as well as the bottom depths at many points in the cross section.

Boat speed, water speed, and depth were used to choose the bin sizes. Boat speeds ranged from 0.1 to 0.5 m/s, and the resulting times to measure a transect was between 6 and 9 minutes. Bin sizes ranged from 2 to 100 cm. Location, water velocity, and boat speed were determined with bottom tracking. Total nine transects were taken for measurement and discharges were within 5% of the mean.

To complete the profiles, velocities and bathymetry were estimated in unmeasured regions of flow. Bathymetry was approximated using a power law fit of

the data points closest to the shore. Velocities at the surface were extrapolated with a power fit over the entire depth. Bottom velocities were also extrapolated from the power fit from the deepest velocity measurement to zero at the bed.

Once the velocity profiles were complete, the transverse mixing coefficient was approximated from the relation  $D = \theta u^* H$  (Rutherford 1994), where  $u^*$  = shear velocity and the coefficient  $\theta$  is calculated as (Deng et al. 2001)

$$\theta = 0.145 + \left(\frac{1}{3520}\right) \left(\frac{U}{u^*}\right) \left(\frac{B}{H}\right)^{1.38} \quad \text{--- (3)}$$

The velocity profiles obtained with the ADCP from Moving Boat measurements were used to estimate the total bed shear velocity,  $u^*$ , applying the law-of-the-wall in the manner suggested by Kostaschuk et al. (2004), i.e., values of  $u^*$  were determined from linear regressions of the form:

$$u = a + b(\ln z) \quad \text{--- (4)}$$

and

$$u^* = kb \quad \text{--- (5)}$$

where  $u$  = flow velocity;  $z$  = height above bed;  $k$  = Von Karman constant (0.41);  $b$  = regression slope coefficient; and  $a$  = regression intercept coefficient.

**4. RESULTS AND DISCUSSIONS**

The computed results of transverse mixing co-efficient ( $D$ ) and longitudinal dispersion co-efficient ( $K_x$ ) of all nine transects are listed in the last two columns of

Table1.

**5. SENSITIVITY ANALYSIS**

A sensitivity and error analysis of the new longitudinal dispersion coefficient equation is conducted for mean values of input and output variables in (2b) and on the assumption that the errors in each input variable are independent. The 9 sets of data of Table 1 give the average values of the channel width, flow depth, velocity, shear velocity, and dispersion coefficient as  $B = 134.7124$  m,  $H = 4.5566$  m,  $U = 0.2403$  m/s,  $u^* = 0.0980$  m/s, and  $K_x = 64.5640$  m<sup>2</sup>/s, respectively.

**Table 2: Sensitivity and Error Analysis of Dispersion Coefficient from eq. (2b).**

X	$\Delta X$	$\Delta K$	Relative error (%)
B	13.4712	7.9693	12.3432
H	0.4557	-1.0609	1.6431
U	0.0240	11.3089	17.5157
$u^*$	0.0098	-3.7441	5.7990

If the error  $\Delta K$  in output longitudinal dispersion coefficient  $K_x$  is defined as the difference between values of  $K_x$  predicted for inputs  $X + \Delta X$  and  $X$ , then the error can be estimated using  $\Delta K = K(X + \Delta X) - K(X)$  where  $\Delta X$  is the error in model input  $X$  denoting the variables  $B, H, U,$  or  $u^*$ . The error could also be expressed in a relative form:  $\Delta K/K$ . The error  $\Delta K$  of the above equation is essentially the deviation sensitivity with  $\Delta X$  being the error. Assuming that each predictor variable is incremented by a constant percentage of 10%, then the errors  $\Delta K$  in dispersion coefficients are computed, as shown in Table 2. Table 2 indicates that the velocity  $U$  is the most sensitive variable among the four input variables; thus, the same change of 10% in  $U$  causes the greatest variation (17.5157 %) in the dispersion coefficient  $K_x$ . The channel width  $B$  is next in importance, followed by shear velocity  $u^*$  and depth  $H$ . Therefore, the prediction accuracy of (2b) depends heavily on the value of velocity  $U$  and its distribution. This means that accurate measurements of flow velocity  $U$  and channel width  $B$  can significantly improve predictions by (2b).

**Influence of Flow and Channel Geometry Change on Dispersion Coefficient**

Table 1 illustrates the variability of the dispersion coefficient in transect of the stream. Actually,  $K_x$  and  $D$  changes even in the same stream with flow and hence water level. Eq. (2b) can be recast by using the Manning formula and shear velocity expression

$$K_x = \frac{0.15}{8D} \left(\frac{1}{n^2} \sqrt{\frac{S}{g}}\right) B^{\frac{5}{3}} H^{\frac{1}{6}} \quad \text{--- (6a)}$$

$$\theta = 0.145 + \left(\frac{1}{3520}\right) \left(\frac{H^{\frac{1}{6}}}{n\sqrt{g}}\right) \left(\frac{B}{H}\right)^{1.38} \quad \text{--- (6b)}$$

Eq. (6) indicates that the longitudinal dispersion coefficient  $K_x$  increases with flow depth  $H$  provided that the water level is maintained in the main channel or the flow discharge is less than the bank-full one. Otherwise, Manning’s roughness coefficient  $n$  may increase significantly once the discharge exceeds the bank-full one, causing the decrease of  $K_x$ . Such a behavior of  $K_x$  with flow is consistent with experimental results (Guymer 1998). The experimental results of both Guymer (1998) and Fischer (1967) have illustrated that compound or more natural cross-sectional geometry channel increases greatly the value of the longitudinal dispersion coefficient. Further investigation is needed to quantify the effects of channel and flow non-uniformities on the longitudinal dispersion coefficient  $K_x$ .

**6. CONCLUSIONS**

Using ADCP velocity and bathymetry measurements to determine the longitudinal dispersion coefficient can reduce the effort and expense of measuring  $K_x$  in many rivers,

increase the understanding of river mixing, and improve the accuracy of predictions of contaminant transport. Further, because the U.S. Geological Survey regularly measures river discharge with ADCPs, the dispersion coefficient can be estimated in many more rivers. We tested the ADCP method with measurements from nine transects at different bend angles of Barak River and the results are encouraging as the results are quite compatible by with the estimates of  $K_x$  from historical dye studies of other natural streams. The ADCP method performs better than two empirical formulas for the dispersion coefficients, and it is at least as accurate as the best formula considered. Error in the method's estimates come from several sources, including applicability of the shear dispersion theory to specific river conditions and limitations of ADCP measurements. An analysis with theoretical velocity profiles shows that the effect of missing data in parts of the profile where the ADCP cannot measure is largest for the most uniform velocity profiles. The explanation is consistent with observations from the comparison of field measurements.

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## 8. NOTATION

The following symbols are used in this technical note:

- $A$  = cross-sectional area ( $m^2$ );
- $a$  = numerical constant (regression intercept coefficient);
- $B$  = top width of river channel (m);
- $b$  = numerical constant (regression slope coefficient);
- $D$  = transverse mixing coefficient ( $m^2/s$ );
- $g$  = acceleration of gravity ( $m/s^2$ );
- $H$  = cross-sectional average depth (m);
- $h$  = local flow depth (m);
- $K$  = longitudinal dispersion coefficient ( $m^2/s$ );
- $k$  = Von Karman constant (0.41)
- $n$  = Manning roughness coefficient;
- $R$  = hydraulic radius (m);
- $S$  = bed slope;
- $U$  = cross-sectional average velocity (m/s);
- $u$  = depth averaged stream-wise velocity (m/s);
- $u'$  = deviation velocity (m/s);
- $u^*$  = shear velocity (m/s);
- $X$  = input variable;
- $y$  = transverse coordinate (m);
- $z$  = height above bed (m)
- $\theta$  = coefficient in approximation of  $D$ .
- $\Delta K$  = error in dispersion coefficient ( $m^2/s$ );
- $\Delta X$  = error in input variable.